# Copula goodness-of-fit testing: An overview and power comparison

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Conference on copulae and multivariate probability distributions in finance – theory, applications, opportunities and problems.

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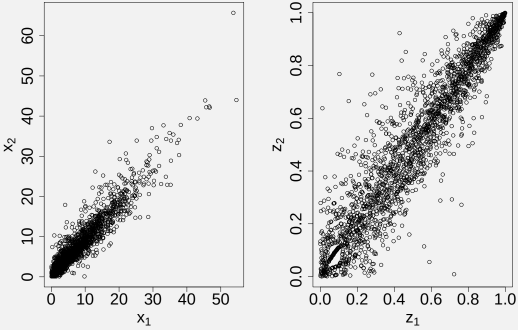
# Motivation

- ▷ Does our model fit the data?



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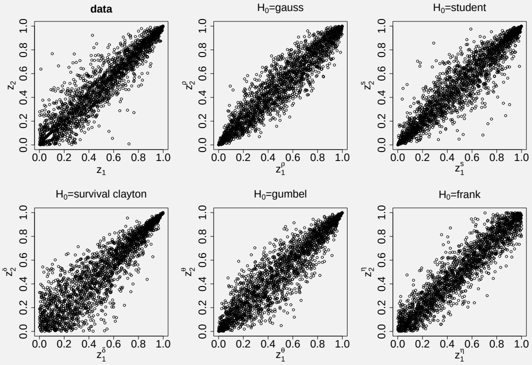


# Outline

# 1.1. Copula GoF testing – Introduction

- $\mathcal{H}_0 : C \in \mathcal{C} = \{C_\theta; \theta \in \Theta\}$ vs. $\mathcal{H}_1 : C \notin \mathcal{C} = \{C_\theta; \theta \in \Theta\}$
- Univariate $\Rightarrow$ Anderson-Darling or QQ-plot,
  Multivariate $\Rightarrow$ fewer alternatives
- Pseudo-observations (normalized ranks):
  $z_j = (z_{j1}, \ldots, z_{jd}) = \left(\frac{R_{j1}}{n+1}, \ldots, \frac{R_{jd}}{n+1}\right), \ j = 1, \ldots, n$
- Limiting distribution of copula GoF test depends on parameter value
- $p$-value estimation via parametric bootstrap procedures
- Focus in literature almost exclusively bivariate
- NOT model selection!

# 1.1. Copula GoF testing – Introduction contd.

- Several techniques proposed:
  - Discretization of unit hypercube (binning)
  - Multivariate smoothing
  - **'Dimension reduction'**

  - Approaches for the testing of one specific copula family
  - **General approaches for the testing of any copula family**

# 1.2. Copula GoF testing – Preliminaries

Rosenblatt's transform:

- ▷ Dependent variables $\Rightarrow$ independent $U[0,1]$ variables, given multivariate distribution
- ▷ $\boldsymbol{v} = \mathcal{R}(z) = (\mathcal{R}_1(z_1), \ldots, \mathcal{R}_d(z_d))$:

$$\begin{aligned} v_1 &= \mathcal{R}_1(z_1) = F_1(z_1) = z_1, \\ v_2 &= \mathcal{R}_2(z_2) = F_{2|1}(z_2|z_1), \\ &\vdots \\ v_d &= \mathcal{R}_d(z_d) = F_{d|1\ldots d}(z_d|z_1, \ldots, z_d). \end{aligned}$$

- ▷ Inverse of simulation (conditional inversion)
- ▷ GoF: $\boldsymbol{v} = \mathcal{R}(z) \Rightarrow$ test $\boldsymbol{v}$ for independence
- ▷ $d!$ different permutation orders

# 1.3.1 Proposed approaches – approach $\mathcal{A}_1$ (1/9)

- $\boldsymbol{v} = \mathcal{R}(\boldsymbol{z}), \quad \boldsymbol{h} = \mathcal{R}(\boldsymbol{v})$
- $W_1 = \sum_{i=1}^{d} \Gamma_V\{V_{(i)}; \alpha\} \cdot \Gamma_H\{H_i; \alpha\}$
- Special case (i): $\sum_{i=1}^{d} \Phi^{-1}(V_i)^2$
- Special case (ii): $\sum_{i=1}^{d} |V_i - 0.5|$
- $S_1(t) = P\{F_1(W_1) \leq t\}, \quad t \in [0,1]$
- CvM statistic:

$$\widehat{T}_1 = n \int_0^1 \left\{ \widehat{S}_1(t) - S_1(t) \right\}^2 \mathrm{d}S_1(t)$$

# 1.3.2. Proposed approaches – approach $\mathcal{A}_2$ (2/9)

- Empirical copula:

$$\widehat{C}(\boldsymbol{u}) = \frac{1}{n+1} \sum_{j=1}^{n} I\{Z_{j1} \leq u_1, \ldots, Z_{jd} \leq u_d\}$$

- CvM statistic:

$$\widehat{T}_2 = n \int_{[0,1]^d} \left\{ \widehat{C}(z) - C_{\widehat{\theta}}(z) \right\}^2 \mathrm{d}\widehat{C}(z)$$

# 1.3.3. Proposed approaches – approach $\mathcal{A}_3$ (3/9)

- Approach $\mathcal{A}_2$ on $\boldsymbol{v} = \mathcal{R}(\boldsymbol{z})$
- CvM statistic:

$$\widehat{T}_3 = n \int_{[0,1]^d} \left\{ \widehat{C}(\boldsymbol{v}) - C_{\perp}(\boldsymbol{v}) \right\}^2 \mathrm{d}\widehat{C}(\boldsymbol{v})$$

# 1.3.4. Proposed approaches – approach $\mathcal{A}_4$ (4/9)

- Cdf of empirical copula (Kendall's dependence function):

$$S_4(t) = P\{C(z) \leq t\}$$

- CvM statistic:

$$\widehat{T}_4 = n \int_0^1 \left\{ \widehat{S}_4(t) - S_{4,\widehat{\theta}}(t) \right\}^2 \mathrm{d}S_{4,\widehat{\theta}}(t)$$

# 1.3.5. Proposed approaches – approach $\mathcal{A}_5$ (5/9)

- Spearman's dependence function:

$$S_5(t) = P\{C_\perp(z) \leq t\}$$

- CvM statistic:

$$\widehat{T}_5 = n \int_0^1 \left\{\widehat{S}_5(t) - S_{5,\widehat{\theta}}(t)\right\}^2 \mathrm{d}S_{5,\widehat{\theta}}(t)$$

# 1.3.6. Proposed approaches – approach $\mathcal{A}_6$ (6/9)

- Shih's test for bivariate gamma frailty model (Clayton):

$$\widehat{T}_{Shih} = \sqrt{n}\left\{\widehat{\theta}_\tau - \widehat{\theta}_W\right\}$$

- Extension to arbitrary dimension:

$$\widehat{T}_6 = \sum_{i=1}^{d-1}\sum_{j=i+1}^{d}\left\{\widehat{\theta}_{\tau,ij} - \widehat{\theta}_{W,ij}\right\}^2$$

# 1.3.7. Proposed approaches – approach $\mathcal{A}_7$ (7/9)

- Inner product of two vectors = 0 iff from the same family

$$Q(z) = \left\langle z - z_{\widehat{\theta}} \middle| \kappa_d \middle| z - z_{\widehat{\theta}} \right\rangle$$

- $\kappa$ a symmetric kernel, e.g. the gaussian kernel:

$$\kappa_d(z, z_{\widehat{\theta}}) = \exp\left\{ -\|z - z_{\widehat{\theta}}\|^2 / (2dh^2) \right\}$$

- Statistic becomes:

$$\widehat{T}_7 = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \kappa_d(z_i, z_j) - \frac{2}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \kappa_d(z_i, z_{\widehat{\theta},j}) + \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \kappa_d(z_{\widehat{\theta},i}, z_{\widehat{\theta},j})$$

# 1.3.8. Proposed approaches – approach $\mathcal{A}_8$ (8/9)

- Approach $\mathcal{A}_7$ on $\boldsymbol{v} = \mathcal{R}(\boldsymbol{z})$
- Statistic:

$$\widehat{T}_8 = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\kappa_d(\boldsymbol{v}_i, \boldsymbol{v}_j) - \frac{2}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\kappa_d(\boldsymbol{v}_i, \boldsymbol{v}_{\widehat{\theta},j}) + \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\kappa_d(\boldsymbol{v}_{\widehat{\theta},i}, \boldsymbol{v}_{\widehat{\theta},j})$$

# 1.3.9. Proposed approaches – approach $\mathcal{A}_9$ (9/9)

- Each approach may detect deviations from $\mathcal{H}_0$ differently
- Average of all approaches:

$$\widehat{T}_9 = \frac{1}{9}\left\{\widehat{T}_1^{(i)} + \widehat{T}_1^{(ii)} + \sum_{t=2}^{8}\widehat{T}_t\right\}$$

## 1.3.10. Proposed approaches – summary

- $\mathcal{A}_1$: Rosenblatt's transform; special cases – Breymann, Dias and Embrechts (2003) and Malevergne and Sornette (2003).
- $\mathcal{A}_2$: Copula distribution function – empirical vs. null
- $\mathcal{A}_3$: Rosenblatt's transform and $\mathcal{A}_2$ – empirical vs. independence
- $\mathcal{A}_4$: Cdf of copula distribution function – empirical vs. null
- $\mathcal{A}_5$: Spearman's GoF process
- $\mathcal{A}_6$: Generalized Shih test – for testing the Clayton copula
- $\mathcal{A}_7$: Panchenko's approach – inner product with gaussian kernel
- $\mathcal{A}_8$: Rosenblatt's transform and $\mathcal{A}_7$
- $\mathcal{A}_9$: An average approach – average of $\mathcal{A}_1$–$\mathcal{A}_8$

# 2.1. MC simulations – Test procedure

1) $x \sim n$ samples from the $d$-dimensional $\mathcal{H}_1$ copula with $\theta(\tau)$.
2) $z \sim$ pseudo-observations (normalized ranks)
3) $\hat{\theta} \sim$ estimated parameter of the $\mathcal{H}_0$ copula
4) $\widehat{T}_i \sim$ test statistic $i$ computed under the $\mathcal{H}_0$ copula using $\hat{\theta}$.
5) Repeat steps 1-4 $M$ times with $\mathcal{H}_1 = \mathcal{H}_0$ and $\theta = \hat{\theta} \Rightarrow \widehat{T}^0_{i,m}$
6) $\hat{p} = \frac{1}{M} \sum_{m=1}^{M} \mathbf{1}(\widehat{T}^0_{i,m} > \widehat{T}_i)$
7) $\hat{p} < 5\% \Rightarrow$ reject $\mathcal{H}_0$

# 2.2. MC simulations – Experimental setup

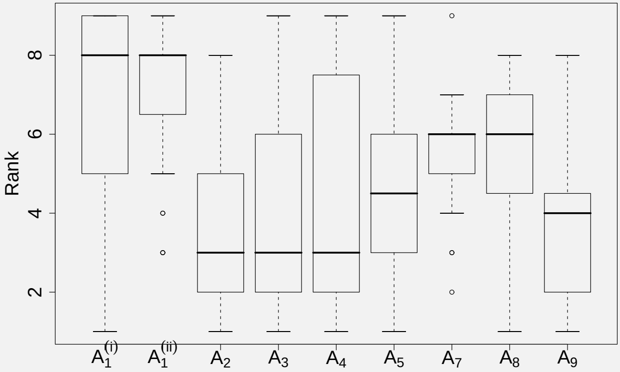
- $\mathcal{H}_0$ copula (4 choices: Gaussian, Student-t, Clayton, Gumbel),
- $\mathcal{H}_1$ copula (4 choices: Gaussian, Student-t ($\nu = 6$), Clayton, Gumbel,
- Kendall's tau (2 choices: $\tau = \{0.2, 0.4\}$),
- Dimension (3 choices: $d = \{2, 4, 8\}$),
- Sample size (2 choices: $n = \{100, 500\}$)
- For each of these $4 \times 4 \times 2 \times 3 \times 2 = 192$ cases $\Rightarrow$ 10,000 repetitions $\Rightarrow$ size/power

# 2.3. MC simulations – Testing the Gaussian copula

Table: Percentage rejections (5% level) of the Gaussian copula ($\tau = 0.2$).

| $d$ | $n$ | $\mathcal{H}_1$ | $\mathcal{A}_1^{(i)}$ | $\mathcal{A}_1^{(ii)}$ | $\mathcal{A}_2$ | $\mathcal{A}_3$ | $\mathcal{A}_4$ | $\mathcal{A}_5$ | $\mathcal{A}_6$ | $\mathcal{A}_7$ | $\mathcal{A}_8$ | $\mathcal{A}_9$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 100 | Ga | *4.8* | *5.5* | *5.0* | *4.7* | *5.2* | *5.4* | – | *4.8* | *4.8* | *5.1* |
| | | St | 1.1 | 4.6 | 7.9 | 8.7 | 6.4 | 5.9 | – | 5.6 | 6.0 | 3.6 |
| | | Cl | 2.9 | 5.1 | 22.1 | 21.1 | 21.0 | 16.3 | – | 7.7 | 7.1 | 11.3 |
| | | Gu | 2.1 | 4.9 | 11.9 | 3.4 | 12.4 | 9.0 | – | 6.0 | 5.9 | 5.2 |
| | 500 | Ga | *4.5* | *4.7* | *5.0* | *4.6* | *4.6* | *5.3* | – | *4.4* | *5.0* | *4.9* |
| | | St | 18.2 | 16.2 | 10.2 | 17.5 | 8.0 | 8.4 | – | 9.3 | 9.8 | 20.2 |
| | | Cl | 2.1 | 5.3 | 73.0 | 72.2 | 71.5 | 56.9 | – | 21.4 | 20.4 | 55.2 |
| | | Gu | 2.5 | 6.1 | 32.9 | 8.9 | 33.0 | 25.7 | – | 11.8 | 12.2 | 21.1 |
| 4 | 100 | Ga | *5.2* | *5.1* | *5.2* | *4.8* | *4.9* | *5.1* | – | *4.8* | *4.9* | *4.9* |
| | | St | 5.1 | 6.3 | 9.3 | 16.0 | 7.9 | 6.5 | – | 6.9 | 6.0 | 7.8 |
| | | Cl | 1.4 | 5.0 | 46.4 | 31.5 | 52.5 | 18.8 | – | 9.4 | 7.2 | 20.9 |
| | | Gu | 1.3 | 3.4 | 12.7 | 0.9 | 41.7 | 56.3 | – | 13.6 | 8.2 | 13.2 |
| | 500 | Ga | *4.8* | *5.9* | *5.8* | *5.8* | *6.2* | *5.4* | – | *5.2* | *4.8* | *5.9* |
| | | St | 98.4 | 74.4 | 18.1 | 49.9 | 12.8 | 13.3 | – | 15.0 | 13.1 | 96.8 |
| | | Cl | 4.1 | 8.9 | 99.0 | 94.9 | 98.1 | 88.8 | – | 38.7 | 20.0 | 94.8 |
| | | Gu | 6.5 | 6.8 | 85.2 | 50.2 | 98.4 | 98.5 | – | 69.8 | 32.1 | 93.8 |

# 2.3. MC simulations – Testing the Gaussian copula

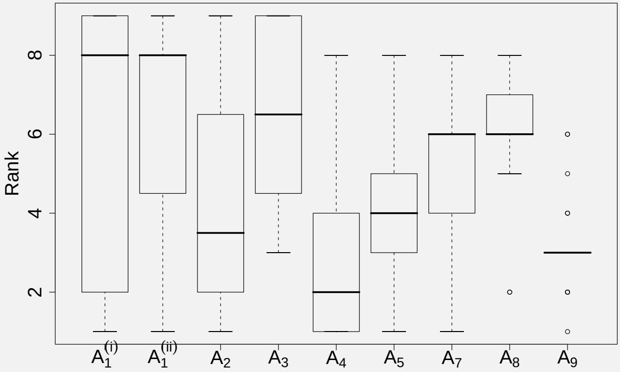


# 2.4. MC simulations – Testing the Student copula

Table: Percentage rejections (5% level) of the Student copula ($\tau = 0.2$).

| $d$ | $n$ | $\mathcal{H}_1$ | $\mathcal{A}_1^{(i)}$ | $\mathcal{A}_1^{(ii)}$ | $\mathcal{A}_2$ | $\mathcal{A}_3$ | $\mathcal{A}_4$ | $\mathcal{A}_5$ | $\mathcal{A}_6$ | $\mathcal{A}_7$ | $\mathcal{A}_8$ | $\mathcal{A}_9$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 100 | Gaussian | 5.6 | 5.7 | 4.5 | 3.9 | 5.0 | 5.3 | – | 5.3 | 4.7 | 5.4 |
| | | Student | 4.4 | 4.9 | 4.7 | 3.9 | 4.9 | 5.2 | – | 5.1 | 4.9 | 4.7 |
| | | Clayton | 5.3 | 5.2 | 19.0 | 10.5 | 20.4 | 17.6 | – | 7.5 | 7.1 | 16.0 |
| | | Gumbel | 4.7 | 5.2 | 9.7 | 5.1 | 10.5 | 7.3 | – | 6.2 | 6.2 | 7.8 |
| | 500 | Gaussian | 5.7 | 5.6 | 5.3 | 4.9 | 5.4 | 5.2 | – | 5.4 | 5.2 | 5.7 |
| | | Student | 4.9 | 4.9 | 4.7 | 4.7 | 5.0 | 5.4 | – | 5.0 | 5.4 | 4.9 |
| | | Clayton | 6.0 | 5.2 | 70.3 | 60.4 | 72.6 | 61.4 | – | 21.9 | 19.6 | 66 |
| | | Gumbel | 5.1 | 5.3 | 28.0 | 18.2 | 29.0 | 18.6 | – | 10.4 | 9.6 | 22.2 |
| 4 | 100 | Gaussian | 6.3 | 6.0 | 4.5 | 4.5 | 5.3 | 5.1 | – | 5.3 | 5.2 | 5.9 |
| | | Student | 4.3 | 4.7 | 4.5 | 4.5 | 5.0 | 5.3 | – | 4.8 | 5.0 | 4.2 |
| | | Clayton | 5.5 | 5.1 | 45.0 | 24.5 | 49.9 | 21.1 | – | 9.5 | 6.8 | 36.8 |
| | | Gumbel | 5.5 | 5.2 | 7.2 | 0.7 | 39.6 | 56.0 | – | 13.2 | 7.6 | 28.3 |
| | 500 | Gaussian | 6.6 | 5.8 | 5.1 | 4.8 | 5.1 | 5.3 | – | 5.4 | 5.3 | 6.1 |
| | | Student | 4.9 | 5.0 | 4.9 | 4.5 | 5.2 | 5.6 | – | 5.1 | 5.1 | 5.3 |
| | | Clayton | 9.0 | 5.5 | 99.1 | 91.3 | 97.7 | 90.2 | – | 35.9 | 21.0 | 97.5 |
| | | Gumbel | 8.0 | 6.7 | 77.1 | 52.6 | 97.6 | 98.0 | – | 65.8 | 31.0 | 96.7 |

# 2.4. MC simulations – Testing the Student copula

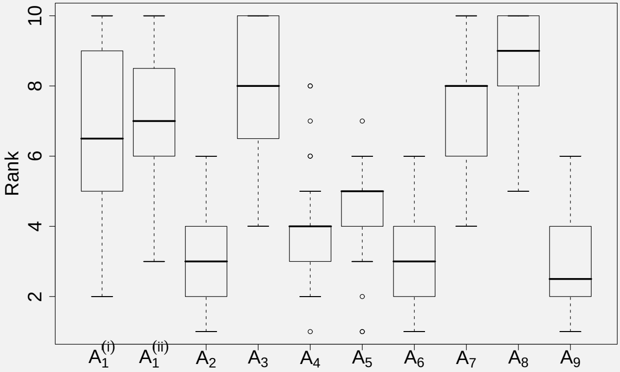


# 2.5. MC simulations – Testing the Clayton copula

Table: Percentage rejections (5% level) of the Clayton copula ($\tau = 0.2$).

| $d$ | $n$ | $\mathcal{H}_1$ | $\mathcal{A}_1^{(i)}$ | $\mathcal{A}_1^{(ii)}$ | $\mathcal{A}_2$ | $\mathcal{A}_3$ | $\mathcal{A}_4$ | $\mathcal{A}_5$ | $\mathcal{A}_6$ | $\mathcal{A}_7$ | $\mathcal{A}_8$ | $\mathcal{A}_9$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 100 | Ga | 7.1 | 7.6 | 21.3 | 6.9 | 22.3 | 14.0 | 20.7 | 7.7 | 6.9 | 20.6 |
| | | St | 8.0 | 8.6 | 24.0 | 8.3 | 23.6 | 16.1 | 17.0 | 8.2 | 7.0 | 21.3 |
| | | Cl | *5.3* | *5.0* | *5.3* | *5.1* | *4.7* | *5.0* | *5.3* | *5.2* | *4.7* | *4.9* |
| | | Gu | 6.3 | 8.8 | 45.8 | 12.5 | 45.6 | 31.1 | 41.0 | 12.9 | 10.2 | 40.5 |
| | 500 | Ga | 18.9 | 13.0 | 77.3 | 46.2 | 69.6 | 54.1 | 85.1 | 23.6 | 20.7 | 68.6 |
| | | St | 25.5 | 22.8 | 80.9 | 35.6 | 73.8 | 64.1 | 68.9 | 24.9 | 22.2 | 76.2 |
| | | Cl | *4.7* | *5.2* | *4.8* | *5.4* | *4.8* | *5.9* | *4.8* | *5.2* | *5.0* | *5.4* |
| | | Gu | 12.9 | 23.4 | 99.2 | 85.9 | 98.1 | 94.3 | 99.1 | 58.6 | 52.6 | 97.4 |
| 4 | 100 | Ga | 11.0 | 10.5 | 37.7 | 2.9 | 39.7 | 39.3 | 50.4 | 9.8 | 7.2 | 48.3 |
| | | St | 26.6 | 20.5 | 48.3 | 17.2 | 37.8 | 41.1 | 37.5 | 10.5 | 8 | 55.9 |
| | | Cl | *4.9* | *4.4* | *4.7* | *4.8* | *5.2* | *4.8* | *4.2* | *5.4* | *5.1* | *4.0* |
| | | Gu | 9.0 | 11.5 | 62.9 | 2.5 | 90.5 | 93.6 | 81.2 | 30.8 | 14.9 | 87.6 |
| | 500 | Ga | 89.8 | 37.7 | 99.5 | 17.3 | 96.5 | 91.1 | 99.9 | 39.1 | 23.6 | 99.4 |
| | | St | 93.5 | 75.7 | 99.9 | 88.8 | 95.1 | 94.0 | 97.6 | 42.6 | 29.7 | 99.9 |
| | | Cl | *5.3* | *4.5* | *5.2* | *5.1* | *5.2* | *5.6* | *4.8* | *5.2* | *5.1* | *4.8* |
| | | Gu | 71.8 | 37 | 100 | 79.6 | 100 | 100 | 100 | 97.7 | 82.9 | 100 |

# 2.5. MC simulations – Testing the Clayton copula

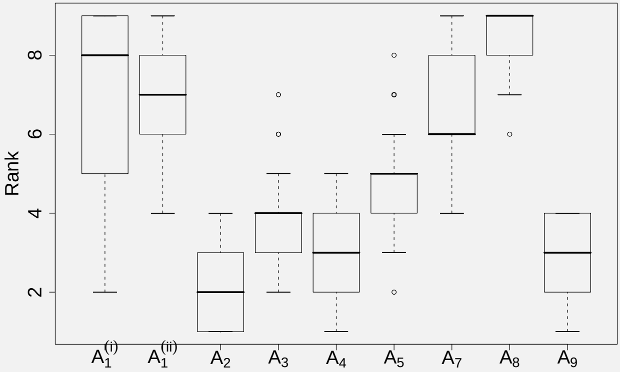


# 2.6. MC simulations – Testing the Gumbel copula

Table: Percentage rejections (5% level) of the Gumbel copula ($\tau = 0.2$).

| $d$ | $n$ | $\mathcal{H}_1$ | $\mathcal{A}_1^{(i)}$ | $\mathcal{A}_1^{(ii)}$ | $\mathcal{A}_2$ | $\mathcal{A}_3$ | $\mathcal{A}_4$ | $\mathcal{A}_5$ | $\mathcal{A}_6$ | $\mathcal{A}_7$ | $\mathcal{A}_8$ | $\mathcal{A}_9$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 100 | Ga | 7.8 | 6.6 | 9.2 | 6.9 | 9.3 | 9.6 | – | 6.3 | 6.1 | 9.8 |
| | | St | 6.6 | 6.8 | 11.1 | 9.9 | 8.9 | 7.0 | – | 5.9 | 5.7 | 9.3 |
| | | Cl | 5.8 | 6.6 | 45.0 | 29.9 | 43.5 | 34.4 | – | 12.1 | 10.3 | 33.0 |
| | | Gu | *5.2* | *5.2* | *5.2* | *5.3* | *5.0* | *5.0* | – | *5.1* | *5.2* | *5.2* |
| | 500 | Ga | 20.9 | 10.2 | 37.5 | 23.2 | 31.1 | 26.9 | – | 12.8 | 11.4 | 33.7 |
| | | St | 17.2 | 11.9 | 40.1 | 33.1 | 26.8 | 17.1 | – | 11.4 | 10.6 | 30.2 |
| | | Cl | 8.9 | 10.6 | 99.5 | 98.0 | 98.4 | 95.5 | – | 58.0 | 52.1 | 97.1 |
| | | Gu | *5.1* | *4.5* | *5.1* | *5.0* | *5.4* | *4.8* | – | *5.3* | *5.1* | *4.9* |
| 4 | 100 | Ga | 7.1 | 13.4 | 54.1 | 44.3 | 52.6 | 25.1 | – | 16.1 | 6.7 | 42.2 |
| | | St | 25.8 | 25.1 | 56.2 | 57.0 | 53.2 | 21.7 | – | 15.2 | 7.5 | 59.0 |
| | | Cl | 3.2 | 15.5 | 89.4 | 85.1 | 97.1 | 82.3 | – | 31.7 | 9.5 | 90.4 |
| | | Gu | *5.1* | *5.0* | *5.3* | *5.3* | *5.2* | *5.1* | – | *5.5* | *5.0* | *4.5* |
| | 500 | Ga | 76.0 | 58.7 | 99.5 | 98.5 | 98.4 | 95.9 | – | 69.4 | 21.0 | 99.4 |
| | | St | 92.1 | 87.9 | 99.1 | 99.8 | 97.8 | 94.4 | – | 67.3 | 29.6 | 100 |
| | | Cl | 34.6 | 65.1 | 100 | 100 | 100 | 100 | – | 98.1 | 55.3 | 100 |
| | | Gu | *5.0* | *4.6* | *4.7* | *5.2* | *5.4* | *5.4* | – | *4.3* | *5.5* | *4.9* |

# 2.6. MC simulations – Testing the Gumbel copula



# 2.7. MC simulations – Conclusions and recommendations

- Nominal levels are well matched
- $\mathcal{A}_2 : \sqrt{n}\{\widehat{C} - C_{\hat{\theta}}\}$ – very good almost always
- $\mathcal{A}_2$ – not so good to detect heavy tails under Gaussian null
- $\mathcal{A}_1$ – good to detect heavy tails under Gaussian null, otherwise poor
- $\mathcal{A}_6$ – very good for testing Clayton null
- $\mathcal{A}_9$ – recommended, in particular if we don't know what kind of deviation we are trying to detect

# 3. Discussion

- 9 approaches considered, including a proposed generalization of Shih's test and two new approaches
- Extensice MC simulations – size/power
- Easier to detect deviations in higher dimensions and with increasing sample size, as expected
- No 'best' approach in general although $\mathcal{A}_2$ comes close
- An average approach such as $\mathcal{A}_9$ recommended
- Disadvantage of $\mathcal{A}_2$, $\mathcal{A}_4$, $\mathcal{A}_5$: computationally intensive when double bootstrap is called for
- Advantage of approaches based on Rosenblatt's transform: computationally efficient
- What about constructions of multivariate dependence?

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