STATISTICAL ANALYSIS OF CREDIT RISK

Topics in Default and Dependence Modelling

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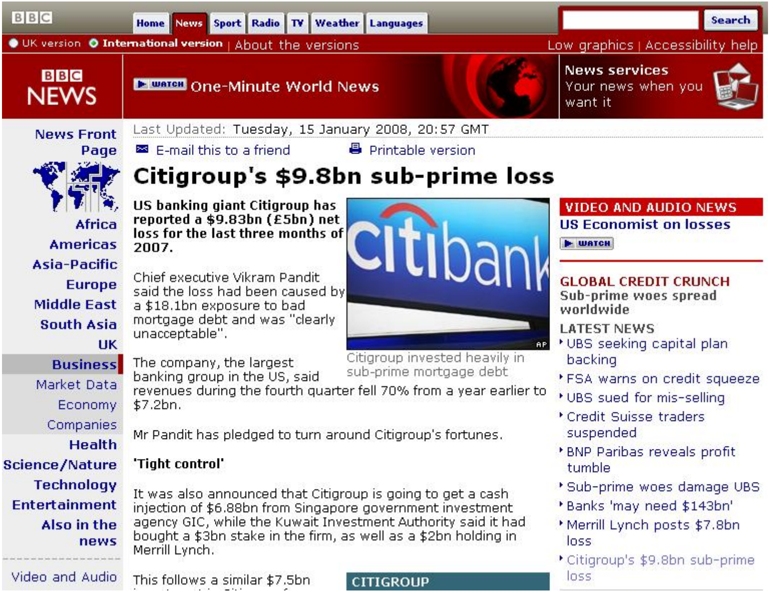

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Citigroup's $9.8bn sub-prime loss

US banking giant Citigroup has reported a $9.83bn (£5bn) net loss for the last three months of 2007.

Citigroup invested heavily in sub-prime mortgage debt

Chief executive Vikram Pandit said the loss had been caused by a $18.1bn exposure to bad mortgage debt and was "clearly unacceptable".

The company, the largest banking group in the US, said revenues during the fourth quarter fell 70% from a year earlier to $7.2bn.

Mr Pandit has pledged to turn around Citigroup's fortunes.

'Tight control'

It was also announced that Citigroup is going to get a cash injection of $6.88bn from Singapore government investment agency GIC, while the Kuwait Investment Authority said it had bought a $3bn stake in the firm, as well as a $2bn holding in Merrill Lynch.

This follows a similar $7.5bn

CITIGROUP

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LATEST NEWS
- UBS seeking capital plan backing
- FSA warns on credit squeeze
- UBS sued for mis-selling
- Credit Suisse traders suspended
- BNP Paribas reveals profit tumble
- Sub-prime woes damage UBS
- Banks 'may need $143bn'
- Merrill Lynch posts $7.8bn loss
- Citigroup's $9.8bn sub-prime loss

Introduction

- **Credit risk** - value of portfolio changes due to unexpected changes in credit quality of issuers or trading partners
- Relates to core activity of most financial institutions; exposure through lending, corporate bond portfolios, otc derivatives, credit derivatives
- Driven by explosive markets for credit derivatives and Basel II active credit risk management is of crucial importance for financial institutions
- Typically depends on
  - Individual default probabilities*
  - Individual losses given default
  - Default dependencies*

Outline

- Paper summary
- Part I: Default modelling
  - Introduction
  - Popular models
  - Papers - purpose, contributions and findings
- Part II: Dependency modelling - Copulae
  - Introduction
  - Definition
  - Using copulae
  - Papers - purpose, contributions and findings

Papers

Part I Default modelling

(i) D. Berg (2007). **Bankruptcy prediction by generalized additive models**. *Appl. Stoch. Models Bus. Ind.*, 23(2), 129–143.

(ii) R. Dakovic, C. Czado, D. Berg (2007). **Bankruptcy prediction in Norway: a comparison study**. *Submitted.*

Part II Dependence modelling - Copulae

(iii) D. Berg, H. Bakken (2007). **A copula goodness-of-fit approach based on the conditional probability integral transformation**. *Submitted.*

(iv) D. Berg (2007). **Copula goodness-of-fit testing: An overview and power comparison**. *Submitted*.

(v) D. Berg, J.-F. Quessy (2007). **Local sensitivity analysis of goodness-of-fit tests for copulas**. *Submitted.*

(vi) D. Berg, K. Aas (2007). **Models for construction of multivariate dependence: A comparison study**. *Submitted.*

Part I

Default modelling

Default prediction

- Any model for the quantification of credit risk relies heavily on a good estimation of default probability
- Binary default variable $Y_i = \begin{cases} 1 & \text{if firm } i \text{ defaults} \\ 0 & \text{else} \end{cases}$
- Look to estimate default probability $p_i = P(Y_i = 1)$
- Two model classes
  - Accounting based*
  - Market based (structural)

Popular models

- Discriminant analysis (Altman's Z-score)
- Neural networks
- Generalized linear models (Ohlson's Z-score)
- Fuzzy logic, support vector machines, ...

Generalized additive models

- Generalized linear models:

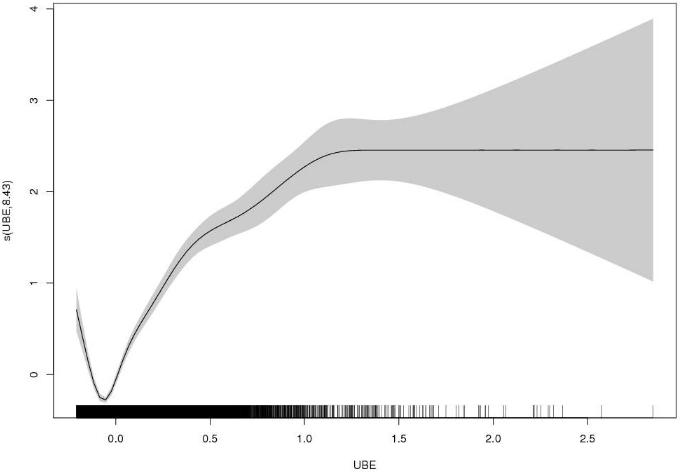
$$\eta = \sum_j \beta_j X_{ij}; \quad p_i = \frac{\exp(\eta)}{1 + \exp(\eta)}$$

- Generalized additive models:

$$\eta = \sum_j f_j(X_{ij}); \quad p_i = \frac{\exp(\eta)}{1 + \exp(\eta)}$$

- f_j estimated through iterative smoothing operations
- Allows for non-linear effects of explanatory variables - estimated non-parametrically

Generalized additive models



Paper I **Bankruptcy prediction by Generalized additive models**
Author: Daniel Berg
Publication details: *Appl. Stoch. Models Bus. Ind.*, 2007

Purpose

- Introduce generalized additive models as a flexible non-parametric alternative for default prediction
- Compare the performance of discriminant analysis, neural networks, generalized linear models and generalized additive models
- Examine prediction power, default horizon, performance depreciation and development sample robustness

Findings

- Generalized additive models significantly outperforms more commonly used models
- Prediction power decreases as default horizon is prolonged
- The performance time-depreciation is evident
- A multi-year model is clearly more robust than models built on the most recent data only

Paper II **Bankruptcy prediction in Norway: A comparison study**
Authors: Rada Dakovic, Claudia Czado, Daniel Berg
Publication details: *Submitted for review*, 2007

Purpose

- Develop models for default prediction in a discrete hazard setting
- Compare generalized linear, generalized linear mixed- and additive models
- Introduce random effects to allow for heterogeneity between industries

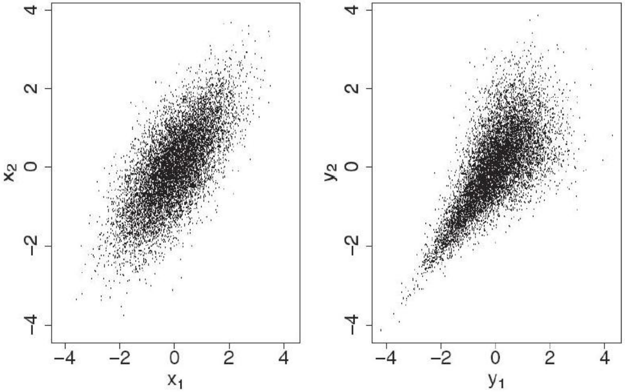
Findings

- Generalized linear mixed seems to perform slightly better than the others
- Hazard instead of static setting does not improve models' performance
- All models outperform celebrated Altman Z-score

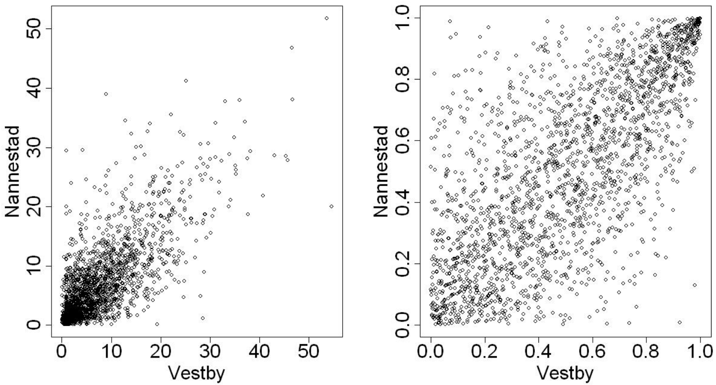
Part II

Dependency modelling - Copulae

Motivation



Motivation



0
10
20
30
40
50
Vestby
Nannestad
0.0
0.2
0.4
0.6
0.8
1.0
Vestby
Nannestad

Brief historical background

- 1940: Hoeffding studies properties of multivariate distributions
- 1959: The word **copula** appears for the first time (Sklar, 1959)
- 1999: Introduced to financial applications (Embrechts et al., 1999)
- 2008: Widely used in insurance, finance, energy, hydrology, survival analysis, etc.

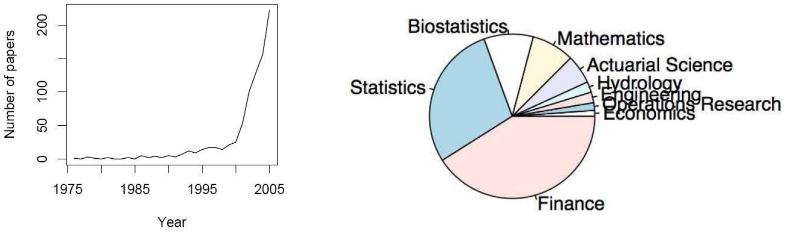


Figure: Based on a survey by Bourdeau-Brien (2007).

Definition & Theorem

Definition (Copula)

A d-dimensional copula is a multivariate distribution function $\mathcal{C}$ with standard uniform marginal distributions.

Theorem (Sklar, 1959)

Let H be a joint distribution function with margins $F_1, \ldots, F_d$. Then there exists a copula $C : [0,1]^d \to [0,1]$ such that

$$H(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d)).$$

Useful results

- A general d-dimensional density h can be expressed, for some copula density c, as
$$h(x_1, \ldots, x_d) = c\{F_1(x_1), \ldots, F_d(x_d)\} f_1(x_1) \cdots f_d(x_d).$$
- Non-parametric estimate for $F_i(x_i)$ commonly used to transform original margins into standard uniform:
$$u_{ji} = \widehat{F}_i(x_{ji}) = \frac{R_{ji}}{n+1},$$
where R_{ji} is the rank of x_{ji} amongst $x_{1i}, \ldots, x_{ni}$.
- u_{ji} commonly referred to as *pseudo-observations* and models based on non-parametric margins and parametric copulas are referred to as *semi-parametric* copulas

Attractive features

- The copula contains all the information about the dependence between random variables
- Copulas provide an alternative and often more useful representation of multivariate distribution functions compared to traditional approaches such as multivariate normality
- Most traditional representations of dependence are based on the linear correlation coefficient - restricted to multivariate elliptical distributions. Copula representations of dependence are free of such limitations.
- Copulas enable us to model marginal distributions and the dependence structure separately
- Copulas provide greater modeling flexibility, given a copula we can obtain many multivariate distributions by selecting different margins
- Any multivariate distribution can serve as a copula
- A copula is invariant under strictly increasing transformations
- Most traditional measures of dependence are measures of pairwise dependence. Copulas measure the dependence between all d random variables

Practical tasks when using copulae

- Parameter estimation
- Model selection
- Model evaluation*
- Simulation

Paper III **A copula goodness-of-fit test based on the conditional probability integral transform**

Authors: Daniel Berg, Henrik Bakken

Publication details: *Submitted for review*, 2007

Purpose

- Examine a proposed copula GoF test based on Rosenblatt's transformation
- Generalize the test to allow for any weight function
- Extend the test to be more robust to inconsistencies
- Examine power of test under Gauss/T mixtures
- Estimate p-values by parametric bootstrap procedure
- Apply the generalized test to financial data

Findings

- Proposed new weights perform much better than original for high dimension and few samples
- Nominal size is kept by bootstrap procedure
- Gaussian copula is strongly rejected for the financial data while T copula performs much better - indicating tail dependence in the data

Paper IV **Copula goodness-of-fit testing: An overview and power comparison**

Author: Daniel Berg

Publication details: *Submitted for review*, 2007

Purpose

- Give an overview of existing GoF tests for copula models
- Carry out extensive simulation studies to compare powers under various conditions
- Examine effect of conditioning order in Rosenblatt's transformation

Findings

- Interesting results from extensive simulations are presented
- Recommendations are made
- Order of conditioning in Rosenblatt's transformation has little influence on the outcome and power of tests based on this approach
- Detailed test procedures are given

Paper V **Local sensitivity analysis of goodness-of-fit tests for copulas**
Authors: Daniel Berg, Jean-François Quessy
Publication details: *Submitted for review*, 2007

Purpose

- Study the asymptotic behavior of several GoF tests for copulas under contiguous alternatives
- Make comparisons between CvM and moment-based tests
- Examine influence of parameter estimator
- Complement asymptotic analysis with extensive simulations for small and medium size samples

Findings

- Asymptotic behavior of several GoF tests for copulas under contiguous alternatives
- Parameter estimator has surprisingly large influence in some cases
- Moment-based tests perform remarkably well
- A new notion of asymptotic relative efficiency is presented

Paper VI **Models for construction of multivariate dependence: A comparison study**

Authors: Daniel Berg, Kjersti Aas

Publication details: *Submitted for review*, 2007

Purpose

- Review models for constructing higher-dimensional copula models
- Compare nested Archimedean models to pair-copula models in terms of interpretation, flexibility and computational complexity
- Fit models and examine GoF on real data

Findings

- Existing models are presented and examined
- Both model classes are applied to two real data sets; precipitation values and stock returns
- Pair-copula constructions are claimed to be easier to interpret, more flexible and in most cases computationally more efficient
- For both applications nested Archimedean models are rejected by the GoF tests while the tests fail to reject the pair-copula constructions